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Experimental investigation on the mechanical performance of helical ceramic springs

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A series of helical ceramic springs were manufactured from MgO partially stabilized zirconia to investigate their mechanical properties. Nine springs were machined from zirconia tubing, initially one inch in length, with a rectangular pitch of 16, 14, or 12 turns per inch. An experimental apparatus that both supports and equalizes the applied loads on springs was developed. The spring deflection versus applied load was measured using an optical sight mounted on a micrometer. Deflection data on each spring were collected, plotted and successfully modeled using Hooke's Law. A more extensive model was used to calculate the shear modulus of rigidity and shear stress. This model incorporates the spring dimensions, pitch, applied load, and deflection and provides insight into the effects of the materials of construction and manufacturing technique on the effective shear modulus of spring. A specific manufacturing effect was observed in the initial deflection resulting from the mass of the spring as the pitch was increased.

Notations

P	Load applied to the spring (kg).
k	Proportionality constant used in Hooke's Law (kg/mm).
f	Resulting deflection based on application of load (mm).
R	Radius of the spring, measured from the central axis of the spring to the center of the coil (mm).
n	Number of active turns in the spring (number of coils in a given length).
G	Shear modulus.
a	Dimension of the spring coil (half the width of coil) (mm).
b	Dimension of the spring coil (half the height of coil) (mm).
D	Diameter of the spring (mm).
G_{eff}	Effective shear modulus.
G_{m}	Shear modulus as a function of the material.
G_{o}	Shear modulus as a function of the construction method imposed.
m	Temperature coefficient (determined from tabulated data).
T	Temperature of the material (°C).
τ	Shear stress (kg/mm^2).
c	Parameter describing spring dimensions (*R*/*b*).

1. Introduction

Few references on helical ceramic springs are available in literature. R. H. Rudolph (1961) reported on helical ceramic springs manufactured from sintered vitreous-bonded alumina [1]. The springs had rectangular helical grooves with diameters of 1.0 to 4.5 inches and wire cross-sections of 0.0625 to 0.25 inches. The springs were determined to have the same bulk properties as the material, alumina in their case, along with unique elastic properties resulting from the manufacturing process. The springs were believed to be valuable in high-temperature applications such as filament supports in vacuum tubes and high temperature relays [1]. Data on these springs were measured up to 1200°F (649°C); however, the author noted that experimental springs had operated up to 2000°F (1094°C).

As few references to ceramic springs exist containing only limited data, models that characterize ceramic spring performance are not readily available. Therefore, we will use modeling equations applicable to metallic springs in an attempt to model the room temperature behaviors of MgO partially stabilized zirconia helical springs. This paper reports on the room temperature data on a new class of zirconia ceramic spring, it successfully applies Hooke's Law to the results and expands the analysis using a more extensive model based on Hooke's Law.

2. Theoretical considerations

In general, springs with a high pitch and thin coils will compress easier than springs with lower pitch and thicker coils. Therefore, springs with the lowest pitch and thickest coil will display the greatest resistance to applied loads. For a spring to obey Hooke's Law, the deflection must vary linearly with the applied load. Hooke's Law is given as:

$$P = kf \tag{1}$$

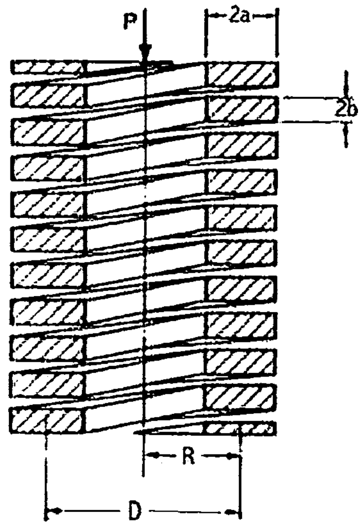

Figure 1 Dimensions of a typical helical ceramic spring [2].

where P is the load applied, f is the deflection, and k is the proportionality constant. A constant based on Hooke's Law that is used in spring design formulas is the spring constant. The spring constant is a parameter that determines the load a spring can support depending on its dimensions.

Fig. 1 shows a drawing of a typical helical spring. The named constants or variables in the figure are related to spring dimensions that are used in further developments below.

For a metallic spring of rectangular cross section, the deflection f, can be determined using the shear modulus G, and a modified equation based on Hooke's Law [3] is given by

$$f = \frac{3\pi PR^3 n}{8Gb^4} \frac{1}{a/b - 0.627[\tan h(\pi b/2a) + 0.004]} \tag{2}$$

where R is the radius of the spring, n is the number of active turns of the spring, a and b are dimensions of the spring as shown in Fig. 1. This equation was modified to account for the construction of the spring. A. M. Wahl (1930) lists an equation that is applicable to a spring with square coils [5].

The equation is given by:

$$f = \frac{44.5PR^3 n}{Gb^4} \tag{3}$$

Equation 3 is applicable for springs with square wire and pitch angles lesser than 12 degrees (A. M. Wahl, 1930), [5]. This equation was further developed for springs with rectangular wires as shown in (2) by W. C. Young [3]. Further reference to the pitch angles has been made by C. J. Ancker, Jr. and J. N. Goodier [9], which supports the results provided by W. Berry [4], to be applicable for springs with small pitch angles between 5 and 20 degrees. Table I provides the pitch angles for the matrix of springs, in addition to the other spring dimensions. Since all the pitch angles fall in the above-mentioned range, we can safely conclude that

TABLE I Summary of spring dimensions

Turns per inch	ID (mm)	*a* (mm)	*b* (mm)	Radius (mm)	Measured pitch angle
12	10.77	1.50	0.50	12.27	5.87°
12	12.34	1.12	0.53	13.46	5.87°
12	12.93	0.94	0.53	13.87	5.87°
14	10.77	1.50	0.36	12.27	5.74°
14	12.34	1.05	0.41	13.40	5.74°
14	12.93	0.94	0.41	13.87	5.74°
16	10.77	1.47	0.27	12.27	5.54°
16	12.34	1.12	0.30	13.40	5.54°
16	12.93	0.97	0.32	13.87	5.54°

Equation 2 is valid for the set of helical ceramic springs, as shown in Fig. 2.

The shear modulus of rigidity, G, also known as torsional modulus of elasticity is a determining factor in spring design formulas, but it is not a true constant in spring design. The shear modulus depends on the mechanical properties of the material being considered, such as the orientation of the grain structure, chemical composition, temperature differentials, and the manufacturing processes [1], and is only applicable if the material obeys Hooke's Law [6]. In the case of machined ceramic springs, the shear modulus depends on the base material, sintering components and theoretical density, operating temperature and method of construction. This can be expressed in the form of an equation as:

$$G_{\text{eff}} = G_{\text{m}} + G_{\text{c}} \tag{4}$$

where G_{eff} is the effective shear modulus, G_{m} is the shear modulus as a function of the material and G_{c} is the shear modulus as a function of the construction method imposed. The shear modulus as a function of temperature can be determined by the equation,

$$G_{\text{m}} = G_{\text{o}}(1 - mT) \tag{5}$$

where G_{o} is the shear modulus at 0°C, T is the temperature of the material given in °C and m is the temperature coefficient that can be determined from the tabulated data. As the temperature of the material increases, the value of the shear modulus typically decreases. Inserting these results into Equation 2, we obtain,

$$f = \frac{3\pi PR^3 n}{8(G_{\text{o}}(1 - mT) + G_{\text{c}})b^4} \times \frac{1}{a/b - 0.627[\tan h(\pi b/2a) + 0.004]} \tag{6}$$

The shear modulus data are available for MgO partially stabilized zirconia from Coor's Ceramics and can be used without further verification [7]. The value listed by Coor's Ceramics for the shear modulus of the material, and consequently, the shear modulus at 0°C, was 7672.5 kg/mm^2. The other mechanical properties for the material can be obtained at www.coorstek.com. Since G_{eff} is determined experimentally and the thermal coefficient, m, is small, the results reported are at

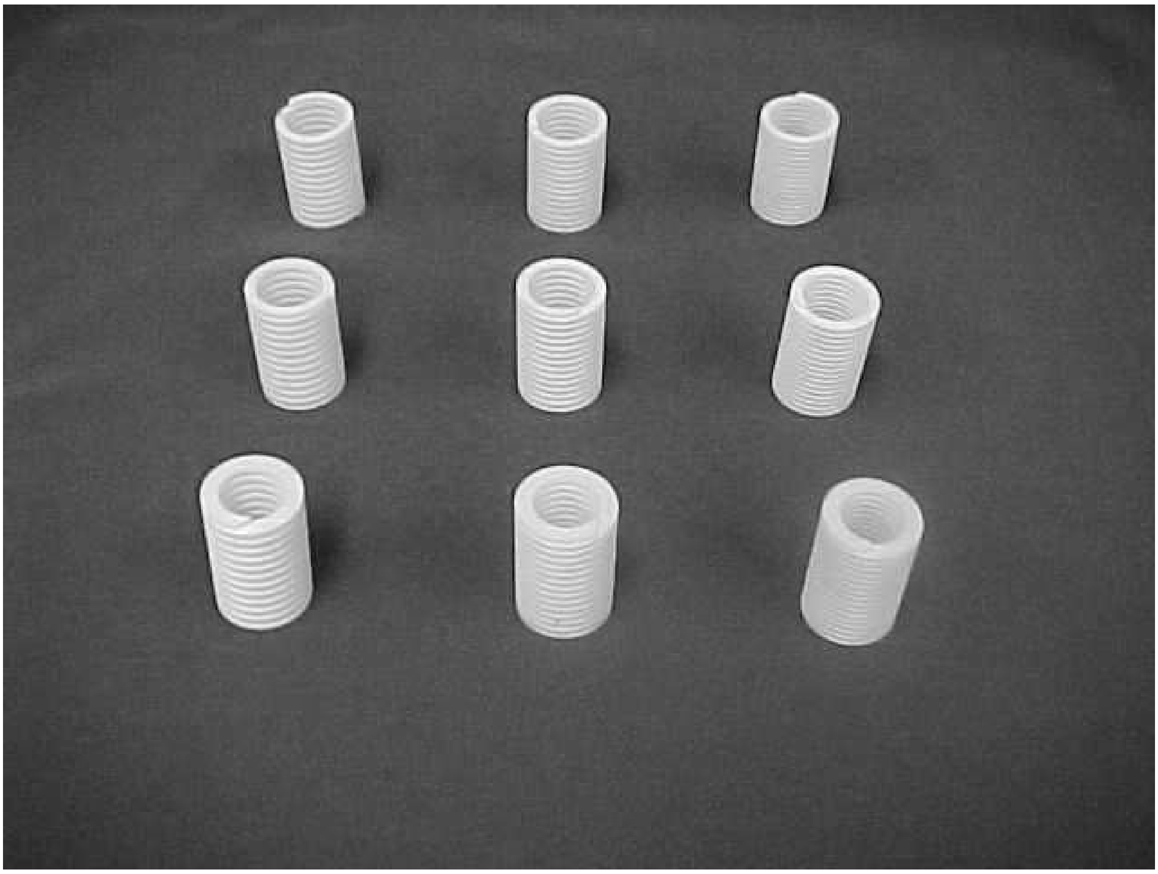

Figure 2 Ceramic springs arranged according to pitch and coil thickness.

the room temperature and the effects of temperature are neglected in Equation 4. The only unknown parameter in Equation 6 is G_c, the shear modulus as a function of the construction method.

The shear stress in a coil is defined as the instantaneous applied shear load divided by the original cross sectional area of the coil over which the load is applied [7]. The equation for the shear stress of a spring of rectangular cross section is given by:

$$\tau = \frac{PR(3b + 1.8a)}{8b^2a^2}\left(1 + \frac{1.2}{c} + \frac{0.56}{c^2} + \frac{0.5}{c^3}\right) \quad (7)$$

and is used without further modification. In Equation 7, the variables refer to the spring dimensions referenced in Fig. 1. The formulas for shear stresses have also been modified according to the spring dimensions and the torque effects. The terms in the parenthesis represent the curvature correction factor, which acknowledges the effects of curvature and direct shear on stress [2]. Wahl, C.J Ancker, Jr. and J. N. Goodier [9] determined the correction factor using derivations of Gohner [8].

3. Equipment and procedures

Machined Ceramics of Bowling Green, Kentucky, manufactured the helical ceramic springs from MgO partially stabilized zirconia tubing obtained from Coor's Ceramic Company, Boulder, Colorado. The designation for this material by Coor's is TTZ. The springs were machined from zirconia tubing with an initial length of 25.4 mm. Tubings with three different wall thicknesses were machined to make 12, 14 and 16 Turns per inch springs. A matrix of nine springs was thus available to characterize their mechanical properties. Fig. 2 shows a picture of the matrix of nine springs. The outside diameter of all the ceramic springs is 19 mm and the additional dimensions of the spring are given in Table I.

To consistently and reproducibly apply loads and measure the deflections on the ceramic springs, the experimental apparatus shown in Fig. 3 was set up.

A machined lava stone upper support was suspended from a triple beam balance that was used to counter balance the mechanism, such that it imposed no load on the spring prior to the application of calibrated weights. The spring was positioned between the upper and lower support to center the spring within the apparatus. Lava stone was selected because of the possibility of the set up being used for high temperature deflection measurements. A right angle telescope with an alignment reticule (cross hairs) was mounted on a three axis linear positioner. A micrometer positions each axis of the linear positioner. Thus, the positioning and deflection measurements were accurate to the width of the reticule in the optical site measured at 0.0025 mm.

A basic measurement began after the system was counterbalanced and the optical sight adjusted to the reference position. Weight was added to the load support to cause the deflection. The calibrated loads were accurate to 0.0001 kg. Once equilibrium was attained, the micrometer on the vertical axis was adjusted to again position the optical site at the reference position. The change in the micrometer reading before and after this adjustment provided a direct measure of the spring

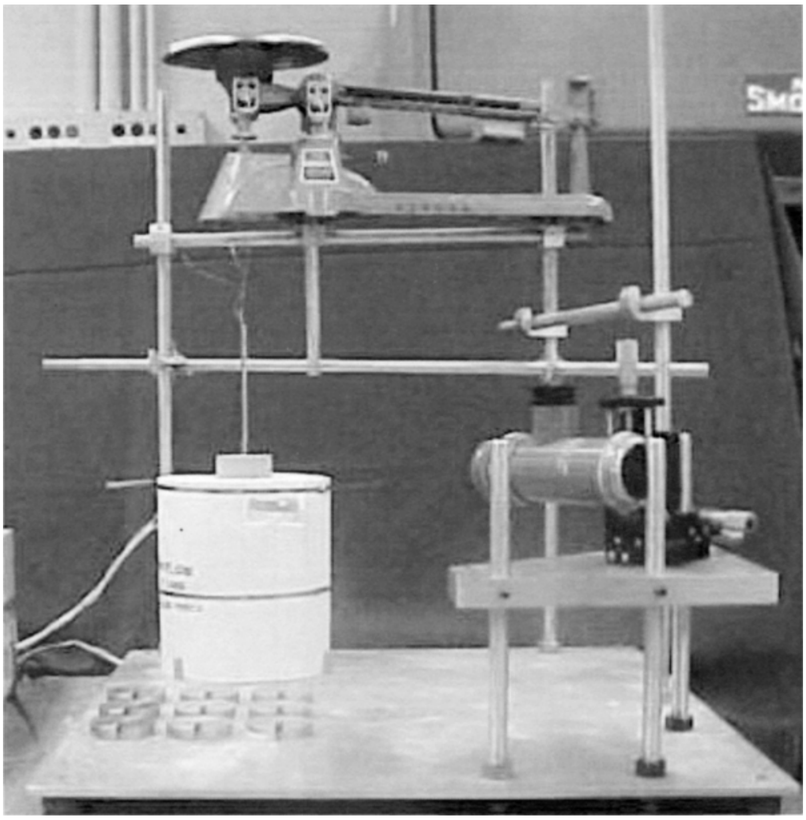

Figure 3 Schematic of spring loading apparatus.

deflection to within 0.0025 mm. This procedure was repeated until the spring was at maximum deflection or fully loaded.

4. Results and discussion

The plot of deflection for each spring tested as a function of applied load is shown in Fig. 4. As seen in the figure, deflection versus load is a linear function for all springs tested, thereby confirming Hooke's Law.

As the pitch decreases, the deflection is decreased, showing that the springs become stiffer. This is expected because the stiffness of the materials would be the limiting factor in case of no turns per inch.

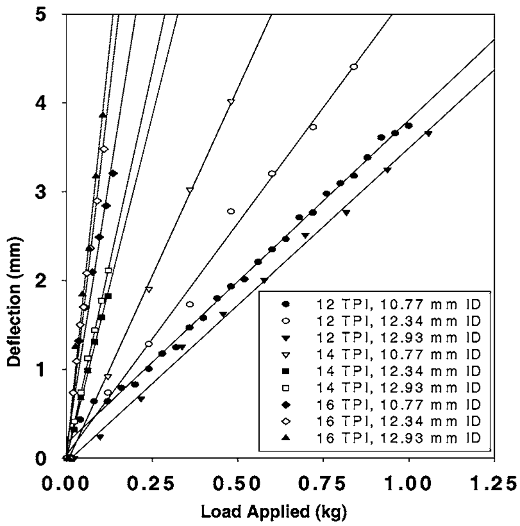

Figure 4 Deflection versus load applied.

The spring index is a dimensionless value that ratios the diameter of the spring by its coil thickness. Using the deflection data, a relationship between the spring constant and the spring index was calculated. Fig. 5 shows the plot of spring constant versus spring index.

As the pitch increased, the slope of the equation used to determine the spring constant decreased. Therefore, springs with higher pitch were less able to resist the same magnitude of the load applied than springs with lower pitch. The average value for the spring constant, based on the models we determined, was measured to be 0.0489 kg/mm. The only available reference in literature to support the analysis for a helical ceramic spring was by Rudolph, who reported a spring constant of 0.0163 kg/mm for a 30 TPI spring with a diameter of 30.163 mm and a coil thickness of 3.175 mm [1]. Since

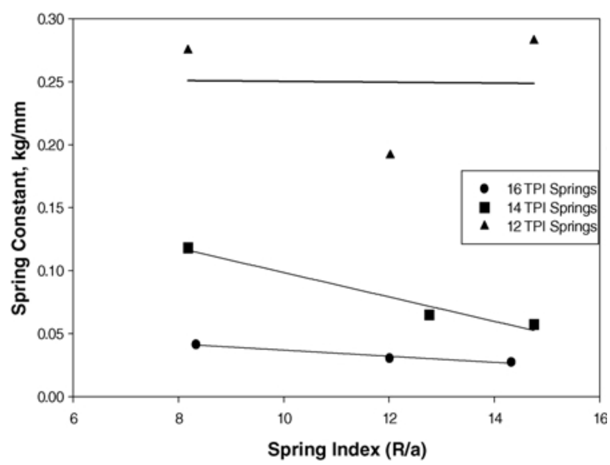

Figure 5 Spring constant versus spring index.

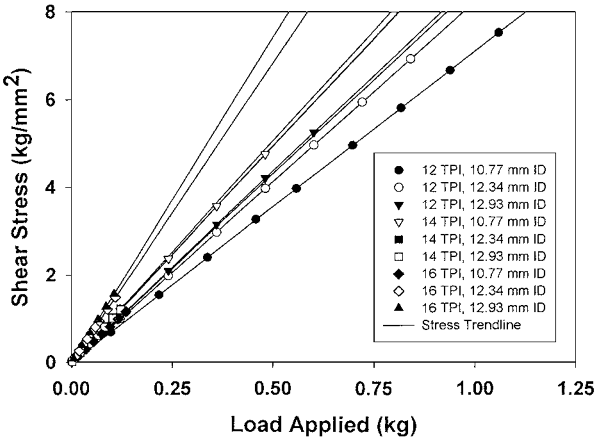

Figure 6 Shear stress versus load applied.

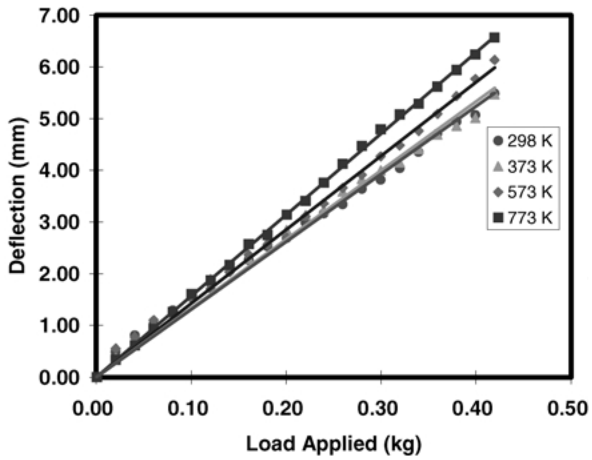

Figure 7 Deflection versus load at increasing temperatures.

both the values for the spring constant are of the same orders of magnitude, the equations developed for the determination of spring constant can be concluded to be applicable for the matrix of helical ceramic springs.

Since Hooke's Law is applicable, the shear modulus of rigidity and the shear stress can be calculated from Equation 2. A linear relationship for the plot of shear modulus versus spring index implies that the shear modulus G decreases slightly with the increase in the spring index. In other words, regardless of the pitch, as the coil thickness increases, the value for the shear modulus increases. This is consistent with the work of R. H. Rudolph (1961). A linear fit of the data shows that the shear modulus is not a true constant based on the materials but rather contains both the material constants and manufacturing elasticity as suggested by Rudolph.

Fig. 6 shows a plot of the shear stress using Equation 2 versus load. As the pitch and thickness increase, there is a linear increase of the shear stress for the spring. The high pitch springs have large deflections at small loading and therefore do not develop much stress.

The deflection of the surrogate spring displayed linear relationship throughout the increasing temperatures. The Fig. 7 shows a plot of deflection versus load at varying temperatures. As a comparison, the deflection of the spring was plotted as an exponential function of the load applied. Results show that an applied exponential relationship between the deflection and the load is visually identical to the applied linear relationship. Both the plots appear linear suggesting that the exponential relationship can be applied to the data, but for developing a model, a linear equation would be adequate. This data supports the hypothesis that the springs made from zirconia ceramic stabilized with magnesium oxide are capable of linear behavior at increasing temperatures as the load begins to increase.

5. Conclusions

The ceramic springs behave very similarly to metal springs at room temperatures. A preliminary testing of a surrogate spring suggested that the ceramic springs would also behave linearly at elevated temperatures. Further testing of the experimental spring matrix is needed to evaluate their performance at high temperatures. The reason for the use of MgO stabilized zirconia is their ability to operate at high temperatures. This property of MgO stabilized zirconia gives hope and confidence for us to continue our experimental work on ceramic springs at elevated temperatures. The deflection was determined to be linear, thereby affirming that Hooke's law was applicable to the ceramic springs and the equations for the shear modulus and shear stress are applicable. Based on these conclusions, a model for determining the behavior of the springs could be presented based on their microstructure, macrostructure, and the temperature at which they perform.

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